It is seen that longitudinal and lateral cyclic pitch are coupled, interacting controls, and that the control system dynamical behavior is strongly dependent on the initial pitch setting σ_0 through the system parameter K_x.

Discussion

The dynamics of collective pitch changes are governed by Eq. (12) where it is seen that the parameter K_x, the virtual spring constant of the centrifugal force field depends on the inertia difference parameter $(I_z - I_y)$ and the initial pitch setting. Ordinarily z is negligible compared to c, so that this parameter is proportional to $\bar{c}^2\cos 2\sigma_0$. Since σ_0 is of the order of 45° at cruising flight speeds of 250 knots in the propeller state, the character of the dynamics are seen to change from that of a heavily damped oscillator (calculation of typical aerodynamic damping values yield values of the order of 50% of critical) in the hovering helicopter state when σ_0 is a small angle to a cascaded integrator and time constant process.

The dynamics of cyclic pitch change are governed by Eq. (17). Expansion of the system characteristic determinant shows that K_x is the critical parameter. In the hovering helicopter state the transient dynamics are those of a pair of coupled, damped oscillators. As σ_0 approaches 45° in the propeller-rotor state, a divergent oscillation ensues.

It is evident that in the propeller-rotor cruise condition, both collective and cyclic pitch changes should have internal stabilization, the principal component of which would be proportional control action to offset the decreasing trend in K_x with forward speed. In the latter case of cyclic pitch change this is seen to be very important. Other compensation techniques would also be beneficial and would depend on the dynamics of the aircraft itself.

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Technical Comments

Comment on "Derivation of the Thrust Equation from Conservation of Energy"

S. W. Greenwood*

University of Maryland, College Park, Md.

SFORZINI[1] suggests that there are advantages in deriving the thrust equation from energy considerations. The following comments are offered in connection with his paper:

1) Sforzini defines g as the "acceleration resulting from gravity." The energy terms in Eq. (1) and (2) of his paper thus have the dimensions (mass × length) instead of the required dimensions (force × length). Gravitational acceleration is not involved in the thrust equation, however derived.

2) The assumption of inviscid external flow at a uniform pressure p_a leading to a resultant rearward force on the control surface of $F_u + (p_a - p_e)A_e$ is part of the conventional momentum theorem approach, as it is of the energy derivation approach. It has the advantage of focusing attention on the associated definition of drag, i.e. the defined thrust F_u minus the actual forward force delivered by the engine. The drag is thus recognized as the sum of the rearwardly directed force due to the viscosity of the external flow and of the rearwardly directed force due to the gage pressure distribution in the external flow. An important contribution to drag is often made by the gage pressure distribution on the flow boundary upstream of the engine inlet plane, i.e., the additive drag.

3) The inclusion of terms involving f, the fuel-air mixture ratio, has advantages where one wishes to derive the rocket thrust equation from that for an air-breathing engine. In air-breathing engines, however, air bled from the compressor for auxiliary purposes such as turbine-blade cooling closely matches the fuel mass flow rate. The bled air discharges at a low energy level and it is more accurate to account for this loss by neglecting the effect of fuel mass addition than by including it in deriving the thrust equation.

4) Propulsive efficiency is of limited value in propulsion studies. It has a maximum value of unity when the thrust is zero. The over-all efficiency, defined as $(F_u u/\dot{m}_f Q_r)$, where Q_r is the heating value of the fuel, is more useful, since the Breguet range is directly proportional to this quantity. As with all definitions of efficiency it is an energy ratio, however F_u may have been derived.

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*Instructor, Department of Aerospace Engineering. Associate Fellow AIAA.

Reference

[1] Sforzini, R. H., "Derivation of the Thrust Equation from Conservation of Energy," *Journal of Aircraft,* Vol. 7, No. 6, Nov.-Dec. 1970, pp. 538–540.